

# Swaption Implied Volatility





## Swaption Volatility

### Swaption Volatility

- An swaption volatility surface is a four-dimensional plot of the implied volatility of a swaption as a function of strike and expiry and tenor.
- Implied volatilities provide indications of the market's near- and long-term uncertainty about future short- and long-term swap rates.
- Vol skew or smile pattern is directly related to the conditional non-normality of the underlying return risk-neutral distribution.
- Smile reflects fat tails in the return distribution whereas a skew indicates return distribution asymmetry.



# Swaption Volatility

## Volatility Surface Construction

- Arbitrage free conditions may be implicitly or explicitly embedded in the procedure. Typical approaches are
  - Local Volatility Model: a generalisation of the Black-Scholes model.
  - Stochastic Volatility Models: such as SABR, Heston, Levy
  - Parametric or Semi-Parametric Models: such as SVI, Omega
  - Market Volatility Model: directly modeling the implied volatility dynamics
  - Interpolation/Extrapolation Model: interpolating or extrapolating volatility data using specific function forms



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## Arbitrage Free Conditions

- Typical arbitrage free conditions
  - Static arbitrage free condition: Static arbitrage free condition makes it impossible to invest nothing today and receive positive return tomorrow.
  - Calendar arbitrage free condition: The cost of a calendar spread should be positive.
  - Vertical (spread) arbitrage free condition: The cost of a vertical spread should be positive.
  - Horizontal (butterfly) arbitrage free condition: The cost of a butterfly spread should be positive.





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## The SABR Model

- SABR stands for “stochastic alpha, beta, rho” referring to the parameters of the model.
- The SABR model is a stochastic volatility model for the evolution of the forward price of an asset, which attempts to capture the volatility smile/skew in derivative markets.
- There is a closed-form approximation of the implied volatility of the SABR model.
- In the swaption volatility case, the underlying asset is the forward swap rate.



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## The SABR Model (Cont.)

- The dynamics of the SABR model

$$d\hat{F} = \hat{\alpha}\hat{F}^{\beta}dW_1$$

$$d\hat{\alpha} = v\hat{\alpha}dW_2$$

$$dW_1dW_2 = \rho dt$$

$$\hat{\alpha}(0) = \alpha$$

where

$\hat{F}$  the forward swap rate

$\hat{\alpha}$  the forward volatility

$W_1, W_2$  the standard Brownian motions

$\rho$  the instantaneous correlation between  $W_1$  and  $W_2$



# Thank You

Reference:

<https://finpricing.com/lib/EqRangeAccrual.html>